

QUIZ #5 – Solutions

Each problem is worth 5 points

15 points total

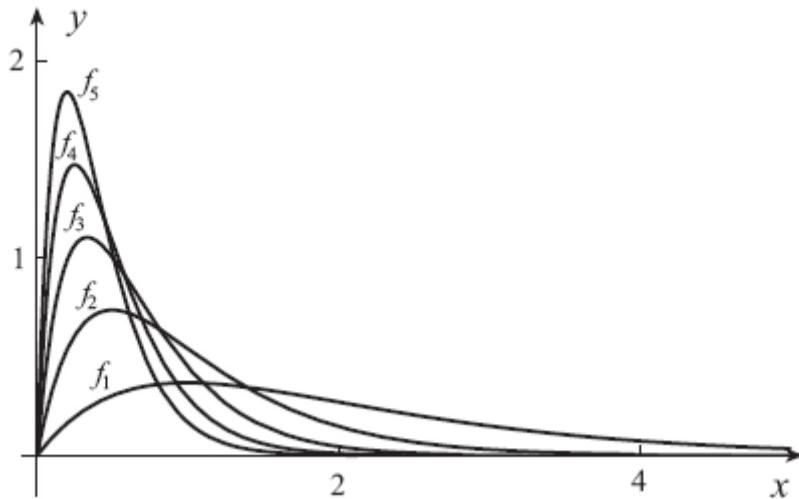
1.

The limit of the sequence $\{n \sin(4/n)\}$ as $n \rightarrow \infty$ is equal to the limit of the function $x \sin(4/x)$ as $x \rightarrow \infty$, provided the limit of the function exists. When we use L'Hôpital's rule,

$$\lim_{n \rightarrow \infty} n \sin\left(\frac{4}{n}\right) = \lim_{x \rightarrow \infty} x \sin\left(\frac{4}{x}\right) = \lim_{x \rightarrow \infty} \frac{\sin(4/x)}{1/x} = \lim_{x \rightarrow \infty} \frac{-(4/x^2) \cos(4/x)}{-1/x^2} = 4.$$

2.

The limit function is $f(x) = \lim_{n \rightarrow \infty} \frac{n^2 x}{e^{nx}}$
 $= \lim_{n \rightarrow \infty} \frac{2nx}{xe^{nx}} = \lim_{n \rightarrow \infty} \frac{2x}{x^2 e^{nx}} = 0.$



3.

Since the radius of convergence is $R = \lim_{n \rightarrow \infty} \left| \frac{n^3 3^n}{(n+1)^3 3^{n+1}} \right| = \frac{1}{3}$, the open interval of convergence is $-1/3 < x < 1/3$.